A Computation Method for Scenario Studies in WDM Network Planning

Abstract —

Purpose – The problem of Wavelength Division Multiplexing (WDM) network planning for a given set of lightpath demands in a mesh topology network is to select lightpath routes and then allocate wavelength channels to the lightpaths. In WDM network planning, a scenario study is to find out the network performance under different lightpath demands and/or different network resource configurations.

Approach – A scenario study must solve a series of related static WDM network planning problems. Each static WDM network planning problem is an optimization problem, and may be formulated as an integer linear programming problem, which can be solved by our proposed Lagrangian relaxation and subgradient methods. We use the Lagrange multipliers that are obtained from previous scenarios as initial Lagrange multiplier values for other related scenarios.

Findings – Our approach dramatically reduces the computation time for related scenarios. For small to medium variations of scenarios, our method reduces the computation time by several folds.

Practical implications – Our method improves the efficiency of a scenario study in WDM network planning. By using our method, many "what-if" type of scenario study questions can be answered quickly.

Originality – Unlike other existing methods that treat each scenario individually, our method effectively use the information of the relation between different scenarios to improve the overall computation efficiency.

Keywords – Optical communication networks, Wavelength Division Multiplexing (WDM) networks, network optimization, resource allocation, routing and wavelength assignment algorithm, scenario studies

Paper type – Research paper

I. Introduction

The wavelength routed Wavelength Division Multiplexing (WDM) technology is the core of broadband networks. It provides huge bandwidth capacity with great network control and management flexibility [1]. In wavelength routed WDM networks, each optical fibre carries multiple wavelength channels. Each wavelength channel in the same fibre uses a distinct wavelength. A WDM switch connects wavelength channels from its incoming fibres to its outgoing fibres. The switching configuration of a WDM switch can be changed through network control or management systems, which means that a lightpath coming into a WDM switch can be switched to a selected outgoing fibre. A lightpath consists of a chain of wavelength channels from a source node to a destination node.

For a given set of static lightpath demands, the WDM network planning problem is a combinatorial optimization problem, which requires a significant amount of computation to obtain the optimal or a near-optimal solution [2]. First of all, because different lightpath demands compete for a common pool of network resources, the resource allocation problem must be solved for the whole set of lightpath demands, not individually. When the number of lightpath demands and network size grow, the computation becomes very difficult. Secondly, for each lightpath demand, it is required to select a route through a mesh topology network and then select wavelength channels along the route. In general, the above two problems are dependent, and need to be solved together to get the optimal solution. These two problems compose the Routing and Wavelength Assignment (RWA) problem.

In WDM network planning, a scenario study is to find out the network performance under different lightpath demands and/or different network resource configurations. A scenario consists of a given set of lightpath demands and a predefined network resource configuration. A scenario study must solve a series of related static WDM network planning problems. For example, lightpath demands are varied to investigate the impact of inaccurate traffic predication, traffic increase or decrease on the network performance. Different network resource configurations are investigated to study the improvement of the network performance with incremental network resource investment or the re-organization of existing network resources. Although various computation methods for a single case WDM network planning were proposed in the literature, most of them are inefficient for scenario studies [2]. They treat each scenario individually, and do not use the relation between scenarios.

In this paper, we present the benefits of using the Lagrangian Relaxation (LR) and subgradient methods in scenario studies for WDM network planning. The LR based method has shown its advantages in computing a near-optimal solution for a single case WDM network planning problem [3, 4]. We use the Lagrange multipliers that are obtained from previous scenarios as initial Lagrange multiplier values for other related scenarios. Our approach dramatically reduces the computation time for related scenarios. For small to medium variations of scenarios, our method reduces the computation time by several folds.

This paper is organized as follows. In Section 2, we present a model and assumptions for a wavelength routed WDM network. In Section 3, we provide an integer linear programming formulation of the static RWA problem. In Section 4, we explain how the LR and subgradient methods are used to solve a single case WDM network planning problem, followed by a proposal of using the Lagrange multipliers that are obtained from previous scenarios to improve the efficiency of scenario study. In Section 5, we present two examples of scenario studies and compare the computation time savings. We conclude this paper in Section 6.

II. NETWORK MODEL AND ASSUMPTIONS

Our network model consists of N nodes interconnected by E links. Each WDM switch is considered as a node in the network model. The set V represents all the nodes in the network. Each link has a pair of fibres, one for each direction. Each fibre has W non-interfering wavelength channels. The link between nodes i and j is denoted by e_{ij}. The c^{th} wavelength channel on e_{ij} is denoted by w_{ijc} ($0 < c \le W$). The set E represents all the links in the network.

We use general mesh network topologies. They are not restricted to any particular pattern. Different nodes may connect to different numbers of other nodes.

We allow wavelength conversion at switches. The number of wavelength converters at different switches varies, and could be zero. Wavelength converters are installed in a share-per-node manner, which means any input or output port may use a wavelength converter, if one is available. If a switch has an available wavelength converter, a lightpath travelling through the switch may use the wavelength converter to change the wavelength; otherwise, a lightpath travelling through the switch must use the wavelength channels with the same wavelength on both its input and output fibres.

In this paper, we use static lightpath demands for WDM network planning purpose. That means lightpath demands all arrive before the planning is conducted, and are all known in advance for the planning. If a lightpath demand is accepted, resources will be allocated to it for constructing a lightpath. A lightpath permanently uses the allocated resources. The set L represents all lightpath demands. Our model allows more than one lightpath being set up between a given node pair. The symbol s_{sdn} denotes the n^{th} lightpath demand between the source-destination node pair (s,d).

We assume all resource costs are known. The total cost of the resources that are consumed by a lightpath is the summation of all the system modules and sub-systems that the lightpath travels through. Since each lightpath uses exactly one transmitter at its source and one receiver at its destination, for simplicity we do not count the costs of transmitters and receivers. For illustration purpose, we only count the resource costs of wavelength channels and converters.

We assume service charges for all lightpath demands are known. At the planning stage, we assume the service charge for a lightpath demand is irrelevant to its consumption of resources. A network operator uses the service charge of a lightpath demand to determine whether accepting the demand is profitable.

III. AN INTEGER LINEAR PROGRAMMING FORMULATION OF THE STATIC RWA PROBLEM

We adopt a penalty-based objective function as in [3], wherein the rejection of demands and the use of network resources are penalized. Since a certain amount of potential revenue is lost when a request is rejected, the rejection penalty equals the amount of its potential revenue. On the other hand, when a request is accepted, its resource consumption is added as a penalty term in the objective function. The resource consumption penalty is the cost of resources used by the lightpath provisioned for the demand.

Our design objective is to minimize the function J, i.e., $\min_{A,\Delta,\Phi}(J)$, where

$$J = \sum_{s_{sdn}\in L}\left[(1-\alpha_{sdn})P_{sdn} + \alpha_{sdn}C_{sdn}\right] \qquad (1)$$

For each demand s_{sdn}, either the penalty of rejecting it (P_{sdn}), or the penalty of using resources (C_{sdn}) to set up a lightpath is added to the objective function (J), depending on s_{sdn}'s admission status α_{sdn}. The value of α_{sdn} is zero, if s_{sdn} is rejected; and α_{sdn} is one, if s_{sdn} is admitted.

In addition to the design variables α_{sdn} ($\forall s_{sdn} \in L$), we introduce the design variables δ_{ijc}^{sdn} ($\forall s_{sdn} \in L$, $\forall e_{ij} \in E$, $0 < c \le W$), representing the use of w_{ijc} by s_{sdn}, and the design variables ϕ_i^{sdn} ($\forall s_{sdn} \in L$, $\forall i \in V$), representing the use of a wavelength converter at node i by s_{sdn}. If w_{ijc} is used by s_{sdn}, δ_{ijc}^{sdn} equals one; otherwise, δ_{ijc}^{sdn} equals zero. If a wavelength converter is used by s_{sdn}, ϕ_i^{sdn} equals one; otherwise, ϕ_i^{sdn} equals zero. We use vector A to denote the acceptance status of all demands, vector Δ to denote their wavelength assignment, and Φ to denote their use of wavelength converters. We use V to denote all the design variables (A,Δ,Φ). For an individual lightpath demand s_{sdn}, we use Δ_{sdn} to denote its wavelength assignment, and Φ_{sdn} to denote its use of wavelength converters. Now we may define the cost of resources C_{sdn} as the cost of using wavelength channels and converters:

$$C_{sdn} = \sum_{e_{ij} \in E} \sum_{0<c\le W} d_{ij} \delta_{ijc}^{sdn} + \sum_{i\in V} o_i \phi_i^{sdn} , \qquad \forall s_{sdn} \in L , \qquad (2)$$

where d_{ij} is the cost of using w_{ijc} and o_i is the cost of using a wavelength converter at node i.

The above static RWA problem must conform to the following constraints.

a) Lightpath continuity constraints:

If a demand is admitted, the lightpath assigned to it has to be continuous along a path between the source-destination pair. Since the assigned lightpath terminates at the two end nodes, we have

$$\sum_{j\in V} \sum_{0<c\le W} \delta_{ijc}^{sdn} - \sum_{j\in V} \sum_{0<c\le W} \delta_{jic}^{sdn} = \begin{cases} \alpha_{sdn} & \text{if } i = s \\ -\alpha_{sdn} & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} , \qquad \forall s_{sdn} \in L \qquad (3)$$

b) Wavelength channel exclusive usage constraints:

$$\sum_{s_{sdn}\in L} \delta_{ijc}^{sdn} \le 1 , \qquad \forall e_{ij} \in E , \ 0 < c \le W \qquad (4)$$

These constraints mean that each wavelength channel can only be used by one lightpath.

c) Transmitter, receiver, and wavelength converter capacity constraints:

$$\sum_{d\in V} \sum_{0<n\le N_{sd}} \alpha_{sdn} \le T_s , \qquad \forall s \in V \qquad (5)$$

$$\sum_{s\in V} \sum_{0<n\le N_{sd}} \alpha_{sdn} \le R_d , \qquad \forall d \in V \qquad (6)$$

$$\sum_{s_{sdn}\in L} \phi_i^{sdn} \le F_i , \qquad \forall i \in V \qquad (7)$$

The number of lightpaths originating from or terminating at a node must be no more than the number of transmitters or receivers at the node. We assume that all transmitters and receivers operate at any wavelength. The number of transmitters at source node *s* is denoted by T_s. The number of receivers at destination node *d* is denoted by R_d. The symbol N_{sd} is the number of lightpath demands between (s,d). The number of used converters at a node must be no more than the number of installed converters at the node. The number of wavelength converters at node i is denoted by F_i.

d) Wavelength conversion constraints:

$$\phi_j^{sdn} = \begin{cases} 1 & \text{if } \exists m,k \in V \text{ and } b \ne a, \delta_{mja}^{sdn} = \delta_{jkb}^{sdn} = 1 \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in V \qquad (8)$$

A wavelength converter at an intermediate node j is used only when different wavelengths are assigned to s_{sdn} for the incoming and outgoing signals at this node.

IV. Computation Method

A. Solve the Static RWA Problem by the Lagrangian Relaxation and Subgradient Methods

We use the Lagrangian Relaxation (LR) and subgradient methods to solve the static RWA problem. By using the LR framework, a Dual Problem (DP) can be derived from the primal problem $\min_V (J)$. A heuristic algorithm is used to obtain a feasible solution to the primal problem from the solution to the Lagrangian DP. The achieved value of the Lagrangian DP is a bound of the objective function in the primal problem. The key of solving the Lagrangian DP is its decomposition into independent sub-problems, whose optimal solutions can be easily obtained. Once the optimal solutions to the sub-problems are computed, we use the subgradient method to solve the Lagrangian DP iteratively. The overall algorithm is illustrated in Figure 1. When the algorithm converges, the optimized Lagrange multipliers are obtained. In addition to the built-in nature of attempting to respect the relaxed constraints in solving the Lagrangian DP, the heuristic algorithm forces the violated constraints to be respected.

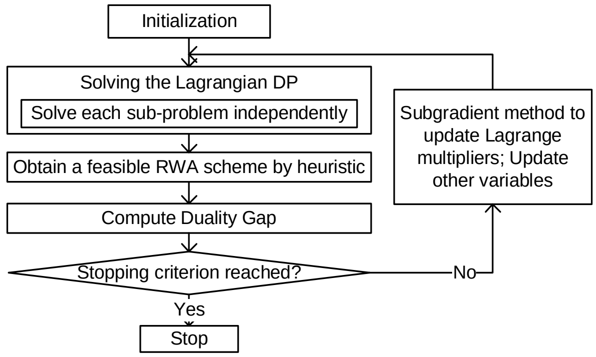

Figure 1. Schematic depiction of the overall algorithm

The Lagrangian DP is derived by relaxing the constraints that represent resource limitations. Lagrange multipliers ξ_{ijc}, π_s, θ_d, and λ_i are introduced in association with the wavelength channel exclusive usage constraints in (4), transmitter, receiver and wavelength converter capacity constraints in (5-7), respectively. We use ξ, π, θ and λ to denote the vectors of Lagrange multipliers (ξ_{ijc}), (π_s), (θ_d), and (λ_i), respectively. We use M to denote all the Lagrange multipliers (ξ,π,θ,λ). The Lagrangian function L is defined as:

$$L(V,M)=J(V)+\sum_{e_{ij}\in E}\sum_{0<c\le W}\xi_{ijc}\left(\sum_{s_{sdn}\in L}\delta_{ijc}^{sdn}-1\right)+\sum_{s\in V}\pi_s\left(\sum_{d\in V}\sum_{0<n\le N_{sd}}\alpha_{sdn}-T_s\right)+\sum_{d\in V}\theta_d\left(\sum_{s\in V}\sum_{0<n\le N_{sd}}\alpha_{sdn}-R_d\right)+\sum_{i\in V}\lambda_i\left(\sum_{s_{sdn}\in L}\phi_i^{sdn}-F_i\right) \quad (9)$$

We define the dual function $q(M)$ as the infimum of $L(V,M)$.

$$q(M)=\min_V[L(V,M)] \quad (10)$$

The Lagrangian DP is $\max_{M\ge 0}[q(M)]$, subject to the constraints in (3) and (8). We use q^* to denote Lagrangian DP's optimal value. The corresponding optimal Lagrange multiplier values are denoted by $M^*=(\xi^*,\pi^*,\theta^*,\lambda^*)$. The optimal value of the Lagrangian DP is a lower bound to the primal problem [5]:

$$q^*(M^*)=\min_V[L(V,M^*)]\le \min_V[J(V)] \quad (11)$$

Two important facts lead to our decomposition of the Lagrangian DP. The first fact is the relations $\delta_{ijc}^{sdn}=\alpha_{sdn}\delta_{ijc}^{sdn}$ and $\phi_i^{sdn}=\alpha_{sdn}\phi_i^{sdn}$. After removing the terms that are independent of the decision variables, the dual function becomes Equation (12). Refer to [3, 4] for the mathematical details. The second fact is that the resource allocation to each lightpath is independent, because the resource usage constraints in (4-7) are relaxed. The complex competition among lightpaths for shared resources does not need to be considered when we allocate resources to individual lightpaths. So, the dual function is composed of the summation of all lightpath-level sub-problems, i.e., $q(M)=\sum_{s_{sdn}\in L}\mathrm{SP}_{sdn}$, where SP_{sdn} denotes the sub-problem that corresponds to s_{sdn}.

$$q(M)=\min_V\left\{\sum_{s_{sdn}\in L}\left[(1-\alpha_{sdn})P_{sdn}+\alpha_{sdn}\left(\sum_{e_{ij}\in E}\sum_{0<c\le W}\delta_{ijc}^{sdn}(\xi_{ijc}+d_{ij})+\sum_{i\in V}\phi_i^{sdn}(\lambda_i+o_i)+\pi_s+\theta_d\right)\right]\right\} \quad (12)$$

The optimal solution to SP_{sdn} is computed by Equation (13). SP_{sdn} corresponds to s_{sdn}'s acceptance or rejection, and the associated RWA problem if it is accepted.

$$\mathrm{SP}_{sdn}(\alpha_{sdn},\Delta_{sdn},\Phi_{sdn})=\min_{\alpha_{sdn}}\left[(1-\alpha_{sdn})P_{sdn}+\alpha_{sdn}\min_{\Delta_{sdn},\Phi_{sdn}}\left(\sum_{e_{ij}\in E}\sum_{0<c\le W}\delta_{ijc}^{sdn}(\xi_{ijc}+d_{ij})+\sum_{i\in V}\phi_i^{sdn}(\lambda_i+o_i)+\pi_s+\theta_d\right)\right] \quad (13)$$

We solve SP_{sdn} in (13) in two steps: lightpath routing, and decision of acceptance or rejection. The first step is to solve the lightpath routing problem:

$$D_{sdn}=\min_{\Delta_{sdn},\Phi_{sdn}}\left\{\sum_{e_{ij}\in E}\sum_{0<c\le W}\delta_{ijc}^{sdn}(\xi_{ijc}+d_{ij})+\sum_{i\in V}\phi_i^{sdn}(\lambda_i+o_i)\right\}, \quad (14)$$

subject to the constraints in (3) and (8) for s_{sdn}. We assign an auxiliary cost $(\xi_{ijc}+d_{ij})$ to w_{ijc}, and an auxiliary cost (λ_i+o_i) to a wavelength converter in node i. The optimal solution is computed by using the modified minimum cost semi-lightpath algorithm in [4].

The second step is to solve the decision problem:

$$\min_{\alpha_{sdn}}\left[(1-\alpha_{sdn})P_{sdn}+\alpha_{sdn}\left(D_{sdn}+\pi_s+\theta_d\right)\right] \quad (15)$$

If P_{sdn} is greater than $(D_{sdn}+\pi_s+\theta_d)$, then reject s_{sdn}. On the contrary, if P_{sdn} is smaller, then accept s_{sdn} (i.e., $\alpha_{sdn}=1$). A tie is broken arbitrarily.

B. Convergence Acceleration in a Scenario Study by Reusing Lagrange Multipliers

An important feature of using the LR and subgradient methods to solve the static RWA problem is that the Lagrange multipliers of two similar scenarios are similar too. The Lagrange multipliers preserve the “neighbourhood property”. When a new scenario is in the neighbourhood of a previous one, the previous Lagrange multipliers produce a good estimate for the new scenario. This property enables reusing Lagrange multipliers to save computation time in scenario studies. To study a new scenario, instead of solving a new optimization process starting from zero Lagrange multipliers, the Lagrange multipliers obtained from the previous scenarios can be reused as initialization points in searching for the optimized Lagrange multipliers. Such approach reduces the time to reach the convergence of the algorithm in a scenario study.

V. EXAMPLES

In the first example, we study a series of scenarios where the lightpath demands change in a fixed network resource configuration and network topology. We use the Pan-European network with 28 nodes and 61 links (shown in Figure 2). The parameters used in the example are $P_{ij0}=1000.0$ for all lightpath demands, $d_{ij}=5.0$ for all links, $F_i=\infty$, $T_i=R_i=18$, $o_i=0$, $t_i=r_i=0$ for all nodes, and $W=16$. We run the heuristic algorithm once every 5 iterations to obtain a feasible solution. The lightpath demands for the two scenarios are shown in Tables 1 and 2. The second scenario is a minor variation of the first one. The variations of the lightpath demands from the first scenario to the second one are highlighted by bold and italic numbers in Table 2.

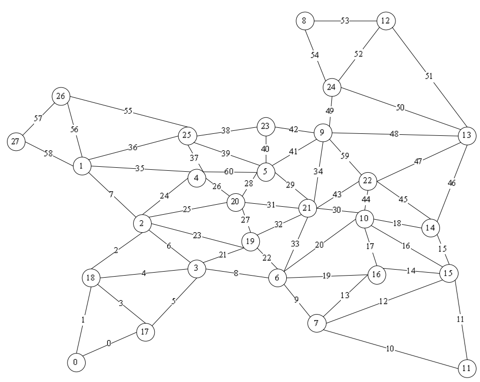

Figure 2. Pan-European network with 28 nodes and 61 links

TABLE 1. INITIAL LIGHTPATH DEMANDS IN THE FIRST NETWORK PLANNING SESSION

0	2	2	2	2	1	2	1	0	0	1	1	0	0	0	0	0	0	0	2	1	1	0	0	0	0	0	0
2	0	2	1	1	2	0	0	0	1	2	0	0	0	0	0	1	2	2	1	1	0	0	0	0	2	0	2
2	2	0	1	2	2	1	0	0	0	1	1	0	0	0	0	1	2	2	2	2	1	0	0	2	0	1	1
2	1	2	0	1	0	2	2	0	1	1	1	0	0	0	1	2	2	1	2	1	1	1	0	1	1	0	0
2	2	2	1	0	2	1	0	1	2	1	0	1	0	0	0	1	0	0	2	2	2	0	2	0	2	0	1
0	2	1	0	2	0	0	2	2	1	2	0	1	1	2	1	0	0	1	0	0	2	1	0	2	0	2	2
2	1	1	0	1	2	0	0	0	0	1	1	0	0	2	0	0	2	2	0	0	0	0	0	0	1	0	2
1	2	1	2	0	0	0	0	0	2	0	0	0	0	0	2	0	1	2	2	0	2	0	0	2	1	1	1
0	0	0	0	0	2	0	1	0	2	1	0	2	2	1	1	0	0	0	1	1	2	2	2	0	1	2	2
0	0	1	0	2	2	1	0	2	0	0	0	2	2	1	0	1	0	1	0	1	2	2	1	0	0	2	2
0	1	0	1	0	1	0	0	1	2	0	2	1	1	2	1	0	0	0	0	2	1	1	1	0	2	0	0
0	1	0	0	0	1	2	1	0	1	1	0	0	0	1	0	2	0	0	1	0	0	0	2	0	1	0	2
0	0	0	1	0	2	1	0	0	2	1	0	0	2	2	1	1	0	0	1	1	2	2	2	1	2	0	0
0	0	0	1	0	0	0	2	2	0	0	0	2	0	0	1	0	0	0	0	0	1	2	2	2	1	0	0
0	0	0	0	0	0	1	2	1	1	0	1	2	1	0	0	1	0	0	1	1	2	2	0	2	0	1	0
0	0	1	1	1	0	0	2	1	0	0	0	1	1	1	0	0	0	0	0	0	2	0	1	1	1	0	0
1	0	0	1	1	1	0	1	0	2	1	2	0	0	2	1	0	0	0	2	0	1	1	0	1	0	0	2
0	2	2	2	2	1	2	1	0	1	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	1	0	0
0	1	2	2	1	0	2	1	0	0	0	0	0	0	0	0	1	2	0	2	2	2	0	0	0	1	0	1
2	1	2	1	2	2	0	0	0	2	0	2	0	0	0	1	0	1	1	0	2	1	0	0	2	0	2	0
2	1	1	0	2	0	1	0	1	0	0	1	1	0	0	2	1	2	2	0	0	1	0	2	0	0	2	0
0	0	0	1	1	0	1	0	2	0	2	0	0	1	2	1	1	1	1	0	0	0	2	0	2	0	0	2
0	2	1	0	0	2	0	2	2	0	2	0	2	2	1	0	2	0	0	1	0	1	0	2	0	2	0	1
0	2	1	0	0	1	0	2	2	1	0	0	2	0	0	2	2	0	0	0	2	0	0	0	2	1	1	0
0	0	0	1	1	0	1	0	0	2	2	0	0	2	2	0	1	0	0	0	1	2	2	2	0	0	0	0
1	2	0	0	2	2	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1	0	0	2	2	0	1	0
1	2	2	1	1	0	2	0	0	0	1	0	0	2	1	0	1	1	2	1	1	1	1	0	1	2	0	2
1	2	2	0	0	2	1	2	0	0	0	1	0	0	0	1	0	1	1	2	0	0	1	2	0	1	0	0

TABLE 2. CHANGED LIGHTPATH DEMANDS FOR THE SECOND SCENARIO

0	2	2	2	2	1	2	1	0	0	1	1	0	0	0	0	0	0	0	2	***2***	1	0	0	0	0	0	0
2	0	2	1	***2***	2	0	0	0	1	2	0	0	0	0	0	1	2	2	1	1	0	0	0	0	2	0	2
2	2	0	1	2	2	1	0	0	0	1	1	0	0	0	0	1	2	2	2	2	1	0	0	2	0	1	1
2	1	2	0	1	0	2	2	0	1	***2***	1	0	0	0	1	2	2	1	2	1	1	1	0	1	1	0	0
2	2	2	1	0	2	1	0	1	2	1	0	1	0	0	0	1	0	0	2	2	2	0	2	0	2	0	1
0	2	1	0	2	0	0	2	2	1	2	0	1	1	2	***2***	0	0	1	0	0	2	1	0	2	0	2	2
2	1	1	0	1	2	0	0	0	0	1	1	0	0	2	0	0	2	2	0	0	0	0	0	0	1	0	***1***
1	***1***	1	2	0	0	0	0	0	2	0	0	0	0	0	2	0	1	2	2	0	2	0	0	2	1	1	1
0	0	0	0	0	2	0	***0***	0	2	1	0	2	2	1	1	0	0	0	1	1	2	2	2	0	1	2	2
0	0	1	0	2	2	1	0	2	0	0	0	2	2	1	0	1	0	1	0	1	2	2	1	0	0	2	2
0	1	0	1	0	1	0	0	1	2	0	2	1	1	2	1	0	0	0	0	2	1	1	1	0	2	0	0
0	1	0	0	0	1	2	1	0	1	1	0	0	0	1	0	2	0	0	1	0	0	0	2	0	1	0	2
0	0	0	1	0	2	1	0	0	2	1	0	0	2	2	1	1	0	0	***0***	1	2	2	2	1	2	0	0
0	0	0	1	0	0	0	2	2	0	0	0	2	0	0	1	0	0	0	0	0	1	2	2	2	1	0	0
0	0	0	0	0	0	1	2	1	1	0	1	2	1	0	0	1	0	0	1	1	2	2	0	2	0	1	0
0	0	1	1	1	0	0	2	1	0	0	0	1	1	***2***	0	0	0	0	0	0	2	0	1	1	1	0	0
1	0	0	1	1	1	0	1	0	2	1	2	0	0	2	1	0	0	0	2	0	1	1	0	1	0	0	2
0	2	2	2	2	1	2	1	0	***0***	0	0	0	0	0	0	0	0	2	2	2	0	0	0	0	1	0	0
0	1	2	2	1	0	2	1	0	0	0	0	0	0	0	0	1	2	0	2	2	2	0	0	0	1	0	1
2	1	2	1	2	2	0	0	0	2	0	2	0	0	0	1	0	***2***	1	0	2	1	0	0	2	0	2	0
2	1	1	0	2	0	1	0	1	0	0	1	1	0	0	2	1	2	2	0	0	1	0	2	0	0	2	0
0	0	0	1	1	0	1	0	2	0	2	0	***1***	1	2	1	1	1	1	0	0	0	2	0	2	0	0	2
0	2	1	0	0	2	0	2	2	0	2	0	2	2	1	0	2	0	0	1	0	1	0	2	0	2	0	1
0	2	1	0	0	1	0	2	2	1	0	0	2	0	0	2	2	0	0	0	2	0	0	0	2	***0***	1	0
0	0	0	1	1	0	1	0	0	2	2	0	0	2	2	0	1	0	0	0	1	2	2	2	0	0	0	0
1	2	0	0	2	2	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1	0	0	2	2	0	***2***	0
1	2	2	1	1	0	2	0	0	0	1	0	0	2	1	0	1	1	2	1	1	1	***0***	0	1	2	0	2
1	2	2	0	0	2	1	2	0	0	0	1	0	0	0	1	0	1	1	2	0	0	1	2	0	1	0	0

We compare the number of iterations required for the convergence of the optimization process. When studying the second scenario, we use two different schemes in initializing the Lagrange multipliers:

Scheme-A (Zero Init): initializing all the Lagrange multipliers to zeros;

Scheme-B (Good Init): initializing the Lagrange multipliers to the obtained optimized Lagrange multipliers from the first scenario.

A dramatic difference on the convergence time is observed between the two initialization schemes (shown in Figure 3). In Scheme-B, the computation almost reached the optimal values after 40 iterations. In contrast, in Scheme-A, the optimization process does not converge to a similar duality gap until after 400 iterations.

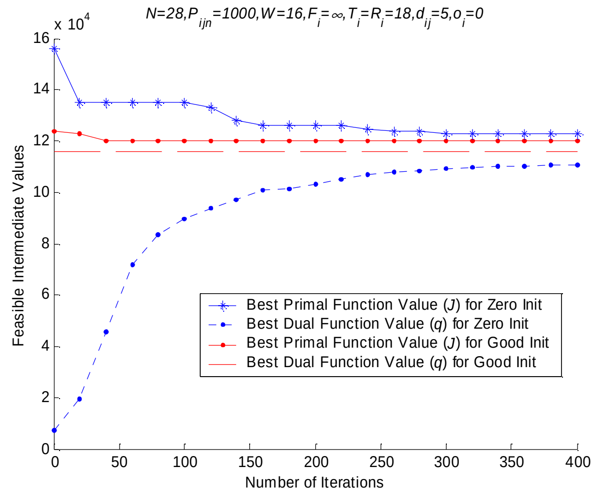

Figure 3. Convergence time comparison between different initialization schemes of Lagrange multipliers

Our extensive simulation results show that the neighbourhood property is very robust. We simulated variations of lightpath demands by 1%, 5%, 10%, and 30%. Our simulation results show that even in the scenarios with relatively large variations of lightpath demands, the previously obtained Lagrange multipliers are still a good estimate for a new scenario, leading to a quick convergence. Another observation is that initializing Lagrange multipliers to zero is generally better than initializing them randomly. Thus, for the first scenario in a series, it is a wise choice to initialize them to all zero.

In a second example, we study a series of scenarios where the network resource configuration changes in a fixed network topology with fixed lightpath demands. We use the network topology shown in Figure 2 and the lightpath demands shown in Table 1. In the original network resource configuration (Case 1), the number of wavelength channels on each link is 16. The achieved design objective is 119788, with a lower bound of 115906. We study three other scenarios. In Case 2, we add two wavelength channels to the six most critical links identified in Case 1, i.e., links 2, 4, 5, 49, 50 and 51. The Lagrange multipliers for the wavelength channels on these links are significantly larger than for those on other links. The achieved design objective is 114859, with a lower bound of 111814. So, adding new resources at critical locations improves the design objective. In Case 3, we add two wavelength channels to six randomly selected non-critical links, e.g., 10, 17, 28, 33, 47 and 57. The achieved design objective is 119780, with a lower bound of 115923. In Case 4, instead of adding resources, we reallocate two wavelength channels from each of the six non-critical links to the six most critical links, i.e., from links 2, 4, 5, 49, 50 and 51 to links 10, 17, 28, 33, 47 and 57. The achieved design objective is 114857, with a lower bound of 111836. The comparisons of convergence time between different initialization schemes for Cases 2-4 are shown in Figures 4-6. The convergences of the three scenarios with good initialization are fairly quick. Table 3 shows the achieved feasible intermediate values of the primal and dual functions for the three cases after a certain number of iterations.

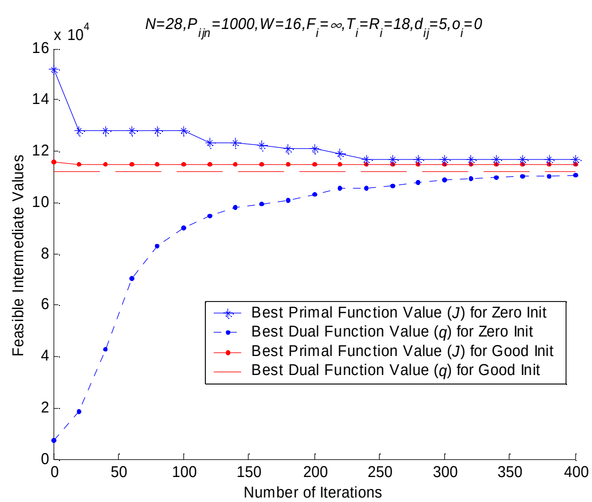

Figure 4. Convergence time comparison between different schemes of Lagrange multiplier initialization for Case 2

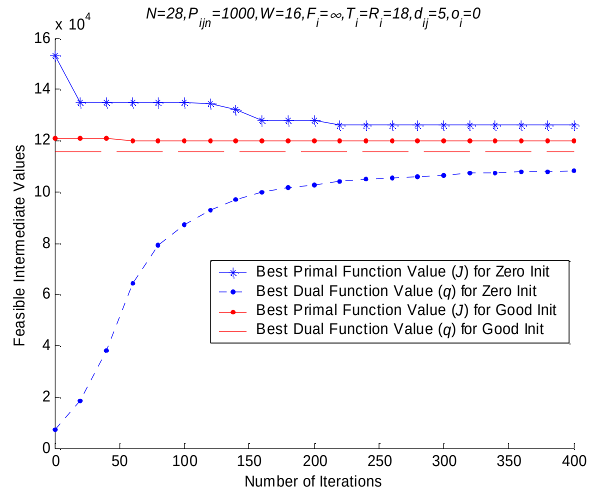

Figure 5. Convergence time comparison between different schemes of Lagrange multiplier initialization for Case 3

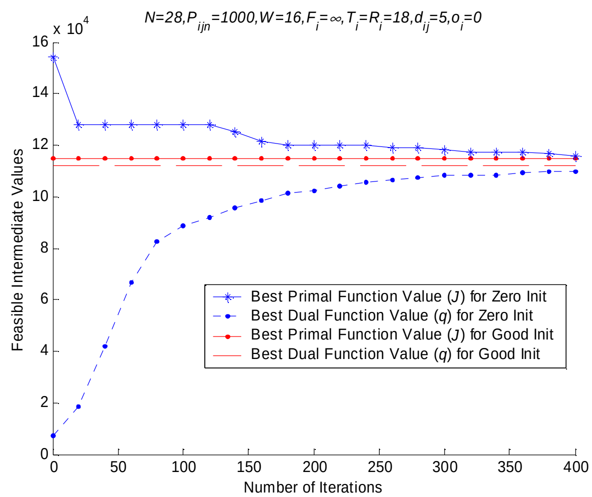

Figure 6. Convergence time comparison between different schemes of Lagrange multiplier initialization for Case 4

TABLE 3. FEASIBLE INTERMEDIATE VALUES OF THE PRIMAL AND DUAL FUNCTIONS FOR THE THREE CASES AFTER A CERTAIN NUMBER OF ITERATIONS

	Case 2		Case 3		Case 4	
Number of iterations	Feasible intermediate value of the primal function	Feasible intermediate value of the dual function	Feasible intermediate value of the primal function	Feasible intermediate value of the dual function	Feasible intermediate value of the primal function	Feasible intermediate value of the dual function
0	115891	111818	120871	115904	114908	111820
20	114899	111818	120871	115904	114908	111820
40	114899	111818	120871	115904	114908	111820
60	114891	111818	119836	115904	114881	111820

VI. CONCLUSIONS

We proposed to use the Lagrangian relaxation and subgradient methods to solve the scenario study problems in static WDM network planning. The optimized Lagrange multipliers that are obtained from previous scenarios are used as initialization points for related scenarios. In this way, the computation time for new scenarios is significantly reduced. Our simulations show that when lightpath demands varies or network resource configuration changes, by reusing previous Lagrange multipliers, the computation time may be reduced by several folds. The superior computation time, together with the good features of the Lagrangian relaxation and subgradient methods, such as being able to provide a performance bound to evaluate the quality of the near-optimal solutions, make our approach attractive in the practical “what-if” types of scenario studies in WDM network planning.

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